6.5 - Work and Energy

Recall Work done by a constant force F over a distance d is

$$W = F \cdot d$$

Def The work done by moving an object along the x-axis from a to b by applying a force of magnitude F(x) is

$$W = \int_a^b F(x)\,dx$$

Hooke's Law A String stretched x units past equilibrium exerts a force of magnitude $-Kx$.

Def Work done against gravity is

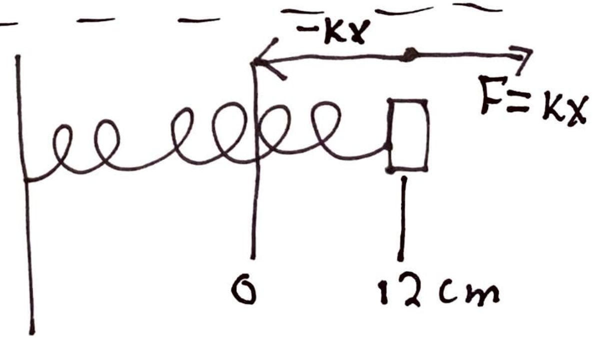
$$W = \int_a^b \mathcal{L}(y)\,dy,$$

where $\mathcal{L}(y) = g \cdot \text{density} \cdot A(y) \cdot \text{vertical distance lifted.}$

Be careful with units!

eg Compute the work required to stretch a spring 12cm past equilibrium. The spring constant is $K = 800\ N/m$.

So



Q1| $12\text{cm} = .12\ m$

Sol

$$W = \int_a^b F(x)\,dx$$

$$= \int_0^{.12} 800x\,dx$$

$$= \left.\frac{800x^2}{2}\right]_0^{(.12)}$$

$$= 400(.12)^2$$

$$W = 5.76\ J$$

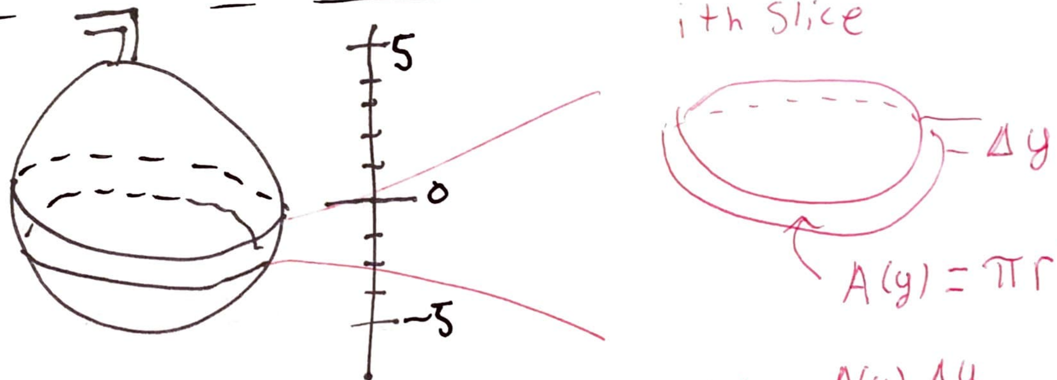
work def

eg

Example 3 in Book

A spherical tank of radius 5 m is filled with water. Calculate the work W performed against gravity in pumping out the water thru a spout 1 m at the top. Density of water is $1000 \frac{kg}{m^3}$.

Sol



$V_i = A(y)\,\Delta y$

mass of V_i

$m_i = \text{density} \cdot V_i$

$F_i = g \cdot m_i$

$$F_i = 9.8\,\frac{m}{s^2} \cdot 1000\,\frac{kg}{m^3} \cdot A(y)\,\Delta y\ m^3$$

Units $\frac{m}{s^2} \cdot \frac{kg}{\cancel{m^3}} \cdot \cancel{m^3} = \frac{kg \cdot m}{s^2}$ ✓

Q2] What are the SI unit of force?

Pg — Chapter 6.5.

A2] $N = \frac{kg \cdot m}{s^2}$

$$F_i = 9800\,\pi r^2\,\Delta y$$

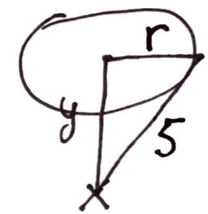
$$W_i = F_i \cdot d_i = F_i\,(6 - y_i)$$

$$W = \sum_{i=1}^{N} F_i \cdot d_i$$

$$W = \int_a^b F(y)\,dy$$

$$W = \int_{-5}^{5} 9800\pi \, r^2 (6-y)\, dy$$

$$= 9800\pi \int_{-5}^{5} (5^2 - y^2)(6-y)\, dy$$



$$= 9800\pi \int_{-5}^{5} 150 - 25y - 6y^2 + y^3\, dy$$

$$= 9800\pi \left[150y - \frac{25y^2}{2} - \frac{6y^3}{3} + \frac{y^4}{4} \right]_{-5}^{5}$$

$r^2 + y^2 = 5^2$

$r^2 = 5^2 - y^2$

$$W = 9,800,000\pi \text{ J}$$