Translational Geometry

eg] Let $\mathcal{T}$ be all transformations of $\mathbb{C}$ which are translations, i.e transformations of the form $Tz = z + b$ for some $b \in \mathbb{C}$. This $(\mathbb{C}, \mathcal{T})$ is called the <u>translational geometry</u> and is a non-euclidean geometry.

Lets verify that $(\mathbb{C}, \mathcal{T})$ is a geometry.

① $id \in \mathcal{T}$: $id : z \to z$

$id(z) = z + (0+0i)$

$id \in \mathcal{T}$.

② We show that for $T \in \mathcal{T}$, then T^{-1} exists and $T^{-1} \in \mathcal{T}$.

Let $T \in \mathcal{T}$, so $Tz = z + b$ for some $b \in \mathbb{C}$.

Define T^{-1} by $T^{-1}z = z - b$. Then $T^{-1}z = z + (-b)$

So $T^{-1} \in \mathcal{T}$.

Next $(T \circ T^{-1})z = T(T^{-1}z) = T(z-b) = (z-b)+b$

$= z + (-b+b)$

$= z + 0$

$(T \circ T^{-1})z = z$

$T \circ T^{-1} = id$

Since T was arbitrary T^{-1} exist and $T^{-1} \in \tau$

for all $T \in \mathcal{T}$.

(3) Let $T_1, T_2 \in \mathcal{T}$, $T_1 z = z + b_1$, $T_2 z = z + b_2$

for some $b_1, b_2 \in \mathbb{C}$. Then $(T_1 \circ T_2)z$

$(T_1 \circ T_2)z = T_1(T_2 z) = T_1(z+b_2) = (z+b_2)+b_1$

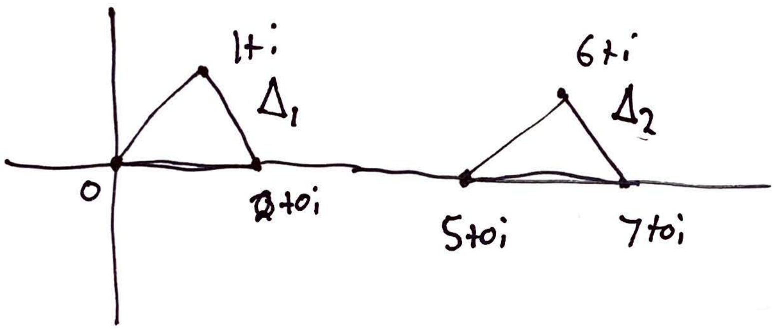
$= z + (b_2+b_1)$. But $b_2 + b_1 \in \mathbb{C}$, so

$T_1 \circ T_2$ is of the form $z+b$, hence

$T_1 \circ T_2 \in \mathcal{T}$.

Finally, the above shows that $\mathcal{T}$ is a transformation group, and $(\mathbb{C}, \mathcal{T})$ is a model of a geometry.

Eg Let Δ_1 be the triangle with vertices $0+0i$, $1+i$, and $2+0i$, and Δ_2 be the triangle with vertices $5+0i$, $6+i$, and $7+0i$.



Claim: $\Delta_1 \cong \Delta_2$ in $(\mathbb{C}, \mathcal{T})$.

To show this claim, we must find a $T \in \mathcal{T}$ so that $T(\Delta_1) = \Delta_2$.

Let $Tz = z + (5+0i)$.

$T(0) = 5+0i$

$T(1+i) = 6+i$

$T(2+0i) = 7+0i$

So $T\Delta_1 = \Delta_2$

thus $\Delta_1 \cong \Delta_2$ in $(\mathbb{C}, \mathcal{T})$.

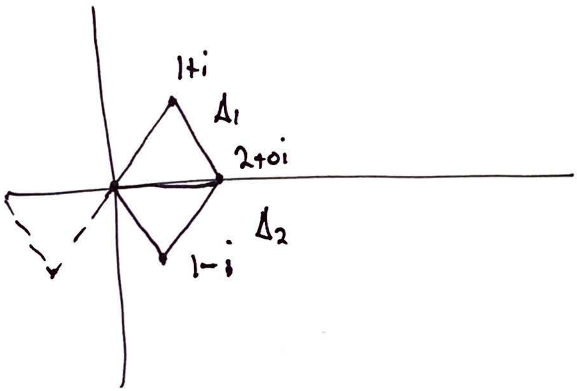
and in

$(\mathbb{C}, E_s)$

$(\mathbb{C}, E)$.

eg] Let Δ_3 be the triangle with vertices

0, $2+0i$, and $1-i$.



There is no $T \in \mathcal{T}$ so that $T\Delta_1 = \Delta_2$

thus $\Delta_1 \not\cong \Delta_2$ in $(\mathbb{C}, \mathcal{T})$.

However $T_2 z = \bar{z}$ has $T\Delta_1 = \Delta_2$

So $\Delta_1 \cong \Delta_2$ in $(\mathbb{C}, E)$.

$\mathcal{T} \subseteq E_s \subseteq E$.