

eg/ Find a Mobius transformation T so that

$$1 \mapsto 3$$

$$i \mapsto 0$$

$$2 \mapsto -1$$

$$Tz = w$$

$$(z, 1, i, 2) = (w, 3, 0, -1)$$

$$\frac{(z-i)(1-2)}{(z-2)(1-i)} = \frac{(w-0)(3+1)}{(w+1)(3-0)}$$

$$\frac{(z-i)(-1)}{(z-2)(1-i)} = \frac{4w}{3w+3}$$

$$\frac{-z+i}{z-z_i-2+2i} = \frac{4w}{3w+3}$$

$$(-z+i)(3w+3) = 4w(z-z_i-2+2i)$$

$$-3zw - 3z + 3iw + 3 = 4wz - 4wz_i - 8w + 8wi$$

$$-3z + 3i = 4wz - 4wz_i - 8w + 8wi + 3wz - 3iw$$

$$-3z + 3i = W [4z - 4zi - 8 + 8i + 3z - 3i]$$

$$-3z + 3i = W [z(4 - 4i) + 3z + 5i - 8]$$

$$-3z + 3i = W [z(7 - 4i) + 5i - 8]$$

$$Tz = W = \frac{-3z + 3i}{z(7 - 4i) + (-8 + 5i)}$$

$$T(1) = \frac{-3(1) + 3i}{1(7 - 4i) + (-8 + 5i)} = \frac{-3 + 3i}{-1 + i} = \frac{3(-1 + i)}{(-1 + i)} = 3$$

$$T(i) = \frac{-3i + 3i}{i(7 - 4i) + (-8 + 5i)} = \frac{0}{12i + 6} = 0$$

$$T(2) = \frac{-3(2) + 3i}{2(7 - 4i) + (-8 + 5i)} = \frac{-6 + 3i}{6 - 3i} = \frac{-1(6 - 3i)}{(6 - 3i)} = -1$$